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Readers' Forum

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Inclusion of Wave Convection Effects in Lighthill's Equation

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THE perceived inability to calculate sound–flow interaction (e.g., wave convection and refraction) via Lighthill's equation[1] has been regarded as a serious flaw. There have been two approaches to resolve this difficulty. Bogey et al.[2] retain the original Lighthill equation and evaluate the right-hand side more exactly via computational fluid dynamics for a simple deterministic example flow. In a different approach, Ribner[3] manipulates the equation into the form of a *convected wave equation*. Its solution has the special feature of morphing into the original Lighthill result outside the refractive "cone of silence" (which it calculates), where that equation has been demonstrated (e.g., Ref. 4) to give good predictive results for jet noise. The two approaches are elaborated and compared in the following. There is a further comparison with procedures based on Lilley's[5] convected wave equation.

Bogey et al.[2] evaluate the principal right-hand-side source term—the "Lighthill (Reynolds stress) tensor"—more accurately, allowing for compressibility, which Lighthill did not. This is done for the specialized scenario of a two-dimensional mixing layer between an upper flow of velocity $0.48c$ and a lower flow of velocity $0.12c$. "The flow is forced at discrete frequencies so that only the sound produced by the first vortex pairings is observed. . . ."[2] Presumably, the flows are separated upstream by a splitter plate, so that the "vortex pairings" would occur downstream of its end. This flow is calculated by large-scale simulation. Because it is deterministic, the instantaneous sound field could be, and is, calculated: it is displayed in a fashion that resembles periodic ripples on a sheet of flowing water. The ripple wave patterns of Figs. 1c and 1d in Ref. 2, particularly the upper ones, show a downstream stretching of the wave pattern by mean flow convection. Furthermore, it is deduced (from amplitude directivity plots that are not shown here, but are available in Refs. 6 and 7) that ". . . there is preferred radiation for large angles from the downstream direction."[2] This is a result of refraction by the mean flow shear: the sound rays are refracted away from the shear layer to enhance the amplitude in an oblique direction. To summarize, Bogey et al.[2] illustrate how two major aspects of sound–flow interaction—convection and refraction—can be determined, by more accurate calculation for a specialized scenario, from the original Lighthill equation. We now discuss the second approach,[3] in which the Lighthill equation is reformulated to have a convected wave operator.

As Lighthill pointed out, wave convection effects—such as refraction by the mean flow shear and scattering by turbulence—are implicit in the right-hand side (in effect via surrogate terms). However, the approximations he made in the terms have the effect of suppressing all of this convective sound–flow interaction. It is this suppression that Bogey et al.[2] address with some success by the injection of compressibility. (However, the inclusion of the mean flow is not the only factor: it was included incompressibly in Ref. 8 without yielding convective refraction.) The suppression of convection is dealt with differently in Ref. 3. The analysis manipulates and repositions the neglected surrogate terms so that the left-hand side becomes a convected wave operator. The surrogate terms are gone from the right-hand side and the approximation $\rho = \text{const}$ therein has been deferred to this later stage. The residual terms on the right-hand side are just Lighthill's final source terms. Physically, these terms are interpreted as radiating into the actual jet flow (in the locally parallel approximation): the theory, as reformulated, is no longer an acoustic analogy.

The result is an internally consistent and complete theory of jet noise that can account quantitatively for all the physically important aspects: generation, convection and refraction, shrouding by other jets, two-microphone cross-correlations, and effects of heating or of foreign gases (the last two via extra terms). The Lighthill source terms (expanded in the form of Ref. 9) have the credibility afforded by the measured source-term microphone cross-correlations of Richarz[10] (as reinterpreted in Ref. 9). (Current speculations about what are the "real" source terms for jet noise quite overlook this experimental support.) The solution effectively morphs into the Lighthill result as the observer angle moves away from the jet axis out of the refractive "cone of (relative) silence." Thus, in the entire region outside of the cone, the simple Lighthill theory is adequate. (It is only near the angle of maximum intensity that there will be some error: the simple Lighthill theory will overpredict by omitting the asymptotic dip into the refractive cone.) This morphing is effected by a squared "normalized" Green's function (frequency dependant) that multiplies the Lighthill solution for the power spectral density. It delineates the refractive cone (via low values) and rises to approximately unity outside the cone.

The widespread misconception that wave convection is important everywhere has led to convected wave alternatives to Lighthill's equation. (This is elaborated in Ref. 3.) The most widely used of these is Lilley's equation.[5] The price, compared with Lighthill's equation, is twofold. First, there is a great increase in complexity but with no demonstrated increase in predictive accuracy outside the refractive cone. Second, there is the lack of a first-order "shear noise" source term on the right-hand side. This is one of the two source terms alluded to in the preceding paragraph that have been experimentally cross-correlated with a microphone signal. Its spectrum can be extracted[11] from jet noise measurements at two angles from the axis along with the "self-noise" spectrum. The two spectra are found to have the same shape, with the second an octave above the first. These two features are predicted by theory[8] based on Lighthill's equation.

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